Bertrand price competition with substance preferences in insurance markets*

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Is the larger substance preferred?

- Substance with independent and homogenous risks
- Solvency II – Value at Risk
- If X and Y are normally distributed, VaR is subadditive
  $VaR_{99.5\%}(X+Y) \leq VaR_{99.5\%}(X) + VaR_{99.5\%}(Y)$
- Can be a motivation to increase the substance
- It may be worth examining the question from a decision theory perspective

Insurance market

Oligopoly vs Perfect competition

- **Turkey**: Kasman – Turgutlu (2008)
- **Netherlands**: J. A. Bikker – M. van Leuvensteijn (2011)
- **Sweden**: Lindmark – Andersson – Adams (2006)
- **Taiwan**: Wang – Tzeng - Wang (2003)
- **Central and Eastern Europe**: Tipurić – Pejić Bach – Pavić (2008)

Most cases seem to support the oligopoly model.

Oligopolies in insurance market

- Bertrand oligopoly – Bertrand paradoxon
- Sonnenholzner and Wambach (2004), Polborn (1998), Wambach (1999) – oligopoly models with positive profit in insurance market
- Villas-Boas and Schmidt-Mohr (1999) – asymmetric information and product differentiation in credit markets

The model

I: number of the insurers

- with the same utility function, u
- with the same wealth, w_0

N: number of the potential customers

- D(p): the probability that a customer will buy at p
- Each customer faces the risk to lose K in an accident with probability q

The model

- Bertrand competition
- The firm that quotes the lowest premium sells to all consumers
- If several insurers quote the same lowest premium, they share the market equally
- $D(p) = \frac{\bar{K}-p}{\bar{K}-qK}$, where $\bar{K}$ is a fixed parameter

Substance preference

The indifferent price

Let $P_n(q,K)$ be the price, when the insurer company is indifferent to do not sell any contracts or to sell n contracts.
$U(w,P_n(q,K),n,q,K) = u(w)$

Substance preference

We say, that u utility function is **substance averse**, iff

$$U(w,P_n(q,K),n+1,q,K) < U(w,P_n(q,K),n,q,K)\ ,\\ \forall n \in \mathcal{Z}^+, q \in (0,1), K \in \mathcal{R}^+\ . \tag{1}$$

If in (1) there is equality, than the utility function is **substance neutral**, if there is greater relation, than it is **substance seeking**.

Substance neutral

The exponential utility function is substance neutral

- $u(w) = -e^{-rw}$, where $r > 0$

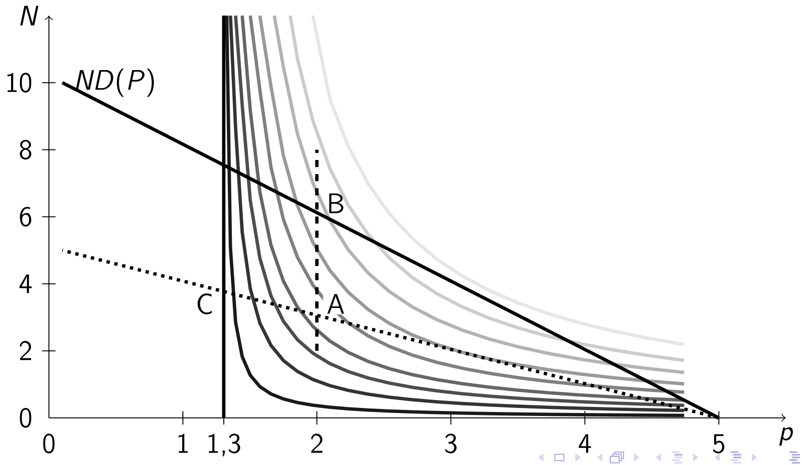
The indifferent price is independent of n

- $P_n(q, K) = \frac{1}{r} ln(qe^{rK} + (1 - q))$

What happens in the market equilibrium?

- No extra utility is avaible in equlibrium

Indifference curves – Substance neutral case



Substance aversion

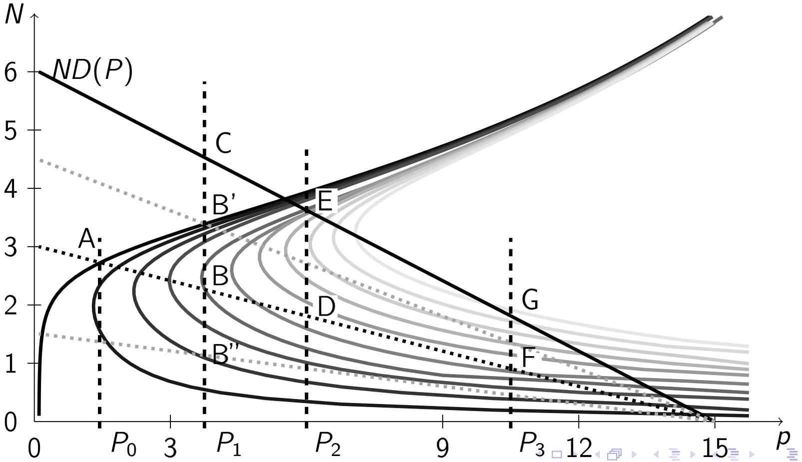
The mixed exponential utility function and the quadratic utility function are substance averse utility functions

- $u(w) = aw - \exp(-rw)$
- $u(w) = w - bw^2,\ b > 0,\ w \leq \frac{1}{2b}$

What happens in the market equilibrium?

- positive extra utility is avaible
- asymmetry in the market is possible

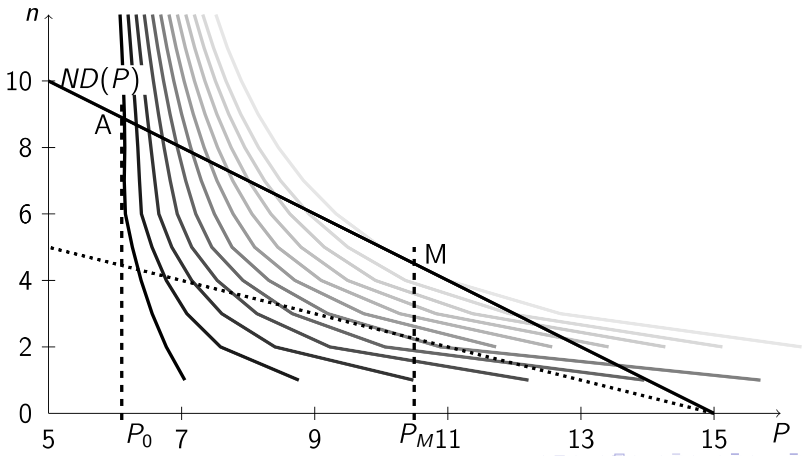
Indifference curves – Substance averse case



Substance seeking

$$u(w) = \begin{cases} 300[-\exp(-1 - \frac{x-100}{140}) + \exp(-1) + \frac{x-100}{1000000}], \\ \text{if } x \leq 100 \\ \\ 750[-\exp(-1 - \frac{x-100}{500}) + \exp(-1) + \frac{x-100}{1000000}], \\ \text{if } x > 100 \end{cases} \tag{2}$$

Indifference curves – Substance seeking case



Proper risk aversion

An undesirable lottery can never be made desirable by the presence of an independent, undesirable lottery (Pratt and Zeckhauser 1987).

Proper risk aversion

Let W, M, L be independent risks. If the u utility function has the proper risk aversion property, then
if

$$Eu(W+L) \leq Eu(W)$$

and

$$Eu(W+M) \leq Eu(W)$$

then

$$Eu(W+L+M) \leq Eu(W+M)$$

Conclusion and further research questions

1. Connection between substance preference and the risk attitude
   - Proper risk aversion usually leads to substance aversion
   - But substance aversion is a more general concept
   - Substance seeking example is an improper utility function
2. Market equlibrium
   - Substance neutral case – no extra utility
   - Substance aversion – extra utility, no unique equilibrium
   - Substance seeking – monopol situation
3. How general are our findings?

Thank you for your attention!

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Risk attitude

Risk Aversion

An agent is risk averse if his or her utility function is concave.

Absolute Risk Aversion

A common measure of absolute risk aversion is the following: $r(x) = \frac{-u''(x)}{u'(x)}$ (Pratt, 1964).

If $r(x)$ is increasing then u is called Increasing Risk Aversion utility function (IARA).
If $r(x)$ is decreasing then u is called Decreasing Risk Aversion utility function (DARA).
If $r(x)$ is constant then u is called Constant Risk Aversion utility function (CARA).

Proper risk aversion - Gollier and Pratt example

$$u(x) = \begin{cases} x & \text{if } x < 100 \\ 50 + \frac{x}{2} & \text{if } x \geq 100 \end{cases}$$

$w_0 = 101$ so $u(w_0) = 100.5$
1. risk 50% $w_1 = 90$ and 50% $w_1 = 115$
2. risk 50% $w_2 = 81$ and 50% $w_2 = 121$

$Eu(1.risk) = 0.5u(90) + 0.5u(115) = 98.75$ so $Eu(1.risk) < u(w_0)$
1.risk is not prefered

$Eu(2.risk) = 0.5u(81) + 0.5u(115) = 95.75$ so $Eu(2.risk) < u(w_0)$
2. risk is not prefered
but $Eu(1.risk + 2.risk) = 96.875 > Eu(2.risk)$

Substance aversion: DARA - Proper risk aversion

If $Eu(W+L) \leq Eu(W)$ and $Eu(W+M) \leq Eu(W)$
then $Eu(W+L+M) \leq Eu(W+M)$.

$W * L * M \preceq W * M \sim W$

Examples for proper risk aversion DARA case
The indifferent price increasing in N - The case of substance aversion

Utility function	$N=1$	$N=2$	$N=3$	$N=5$	$N=10$
$ln(x)$	1.052991	1.052996	1.053002	1.053012	1.053039
$\sqrt{x}$	1.026043	1.026045	1.026047	1.026051	1.026062
$x - \frac{e^{-x/10}}{1000}$	0.121747	0.346822	2.201726	9.789496	18.2315