Basis Risk in Variable Annuities

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Introduction

- **Variable Annuities (VAs)**: hybrid products offered by life insurance companies
  - Popular personal savings/investment vehicles
  - Nearly $2.0 trillion in net assets by mid-2019
  - Largest liability component for U.S. life insurers
  - Policyholders determine investment allocations along large numbers of mutual funds (MFs)
  - VA investment is placed into a separate account
  - Financial guarantees akin to long-term put options
  - Fees are collected to finance the potential shortfall of the account value
  - Hedging liabilities is challenging: long term nature and complex structures
  - Financial Crisis: VAs could cause significant risk mismatch (Koijen and Yogo, 2017)

Research Question

- **Basis risk** increases the difficulties in the hedging of VAs
  - Ideal hedge of the VA liabilities: *short* position in the MFs
  - MFs: typically cannot be shorted and are relatively illiquid
  - Alternative hedging instruments: equity futures, options, interest swaps, and ETFs
  - Basis risk in VAs: discrepancy in returns between MFs and hedging instruments
  - Jeopardizes the hedging efficiency (Milliman, 2016; Trottier et al., 2018)
  - Threatens the long-term financial well-being of U.S. life insurers
- First comprehensive empirical analysis to quantify magnitude of basis risk faced by U.S. VA providers
  - 1,890 VA-underlying mutual funds with 470 ETFs and 5 instrument sets

Preview of Methodology and Findings

- Basis risk measure
  - The proportion of risk (i.e., standard deviation) of the fund's return that cannot be eliminated through hedging (Ederington, 1979)
  - External information regarding investment strategies: Lipper Objective Code (LOC)
  - Machine learning method in addition to traditional fund mapping techniques
- VA-underlying mutual funds: **basis risk is substantial and pervasive**
  - **26.6%** on average across 1,890 VA-underlying mutual funds
  - 25 out of 27 Lipper Objective Code classes have a median basis risk above 20%
  - Index funds have substantially lower basis risk

Literature Review

- Variable Annuities
  - Pricing of the VA guarantees: Bauer et al. (2008), Dai et al. (2008), etc
  - Investor's exercise behavior: Knoller et al. (2016), Moenig and Bauer (2016), etc
  - Hedging strategies of VAs: Kling et al. (2011), Augustyniak and Boudreault (2017), etc
- Basis risk in VAs: Ankirchner et al. (2014) and Trottier et al. (2018a; 2018b)
  - Existing studies: severely limited in the number/variety of MFs and hedging instruments
- Fund Mapping Techniques
  - Fund style analysis: Sharpe (1988; 1992) Bodson et al. (2010), etc
  - Used in the hedging of VAs: Gan and Valdez (2017) and Fan et al. (2018)

Fund Mapping Process

- Identify instruments as proxies for each VA-underlying mutual fund (Sharpe, 1992)
  - Monthly return of the VA-underlying mutual fund j in month s: FR_{js}
  - Monthly return of the instrument i in month s: IR_{is} $(i = 1, ..., M)$
  - Monthly return of the risk-free asset in month s: Rf_s

$$FR_{js} - Rf_s = \sum_{i=1}^{M} \beta_i (IR_{is} - Rf_s) + \epsilon_s \qquad (\text{for } s = 1, \ldots, N)\,. \tag{1}$$

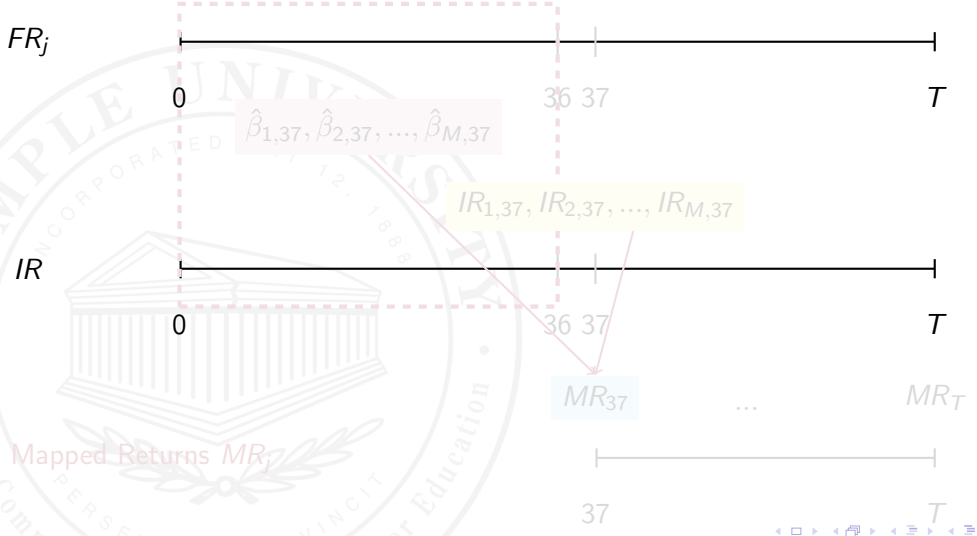
Fund Mapping Process

- Identify instruments as proxies for each VA-underlying mutual fund
  - The length (in months) of the rolling window period: N

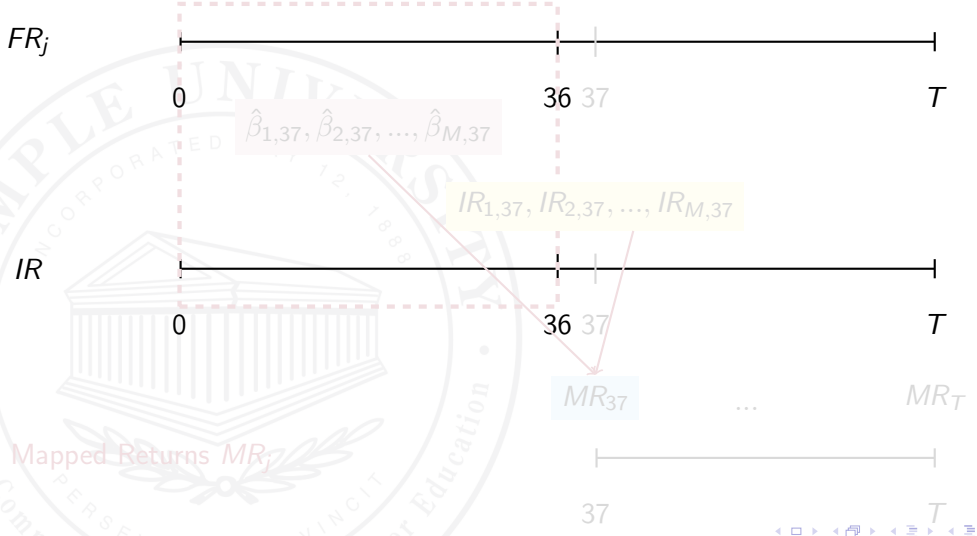
$$FR_{js} - Rf_s = \sum_{i=1}^{M} \beta_{it}(IR_{is} - Rf_s) + \epsilon_s \qquad (\text{for } s = t-N, \ldots, t-1)\,. \tag{2}$$

  - The weight of instrument i for month t: β_{it}
  - The weight of risk-free asset for month t: $1 - \sum_{i=1}^{M} \beta_{it}$
- Rolling window period: $N = 36$ **months** (Buetow et al., 2000; Atkinson and Choi, 2001)

Rolling Window Period



Rolling Window Period



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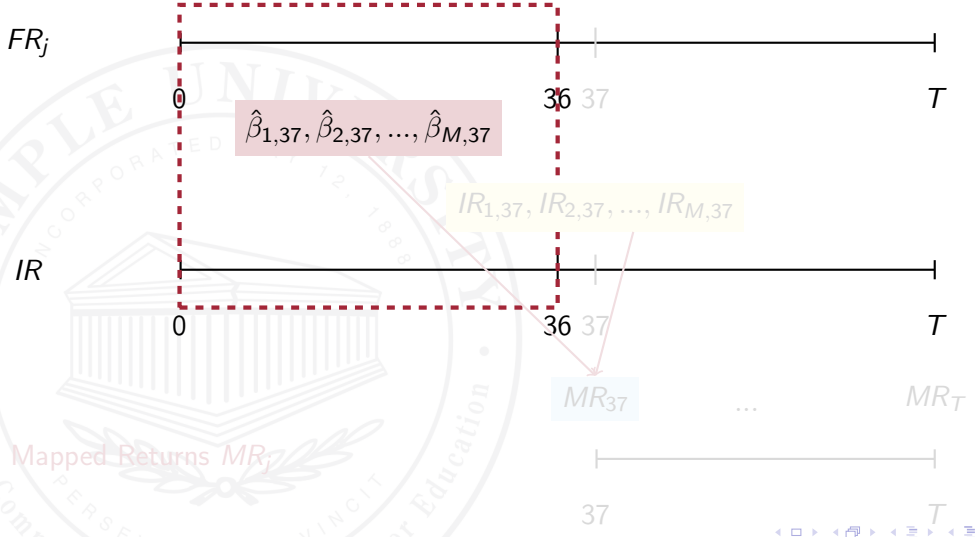

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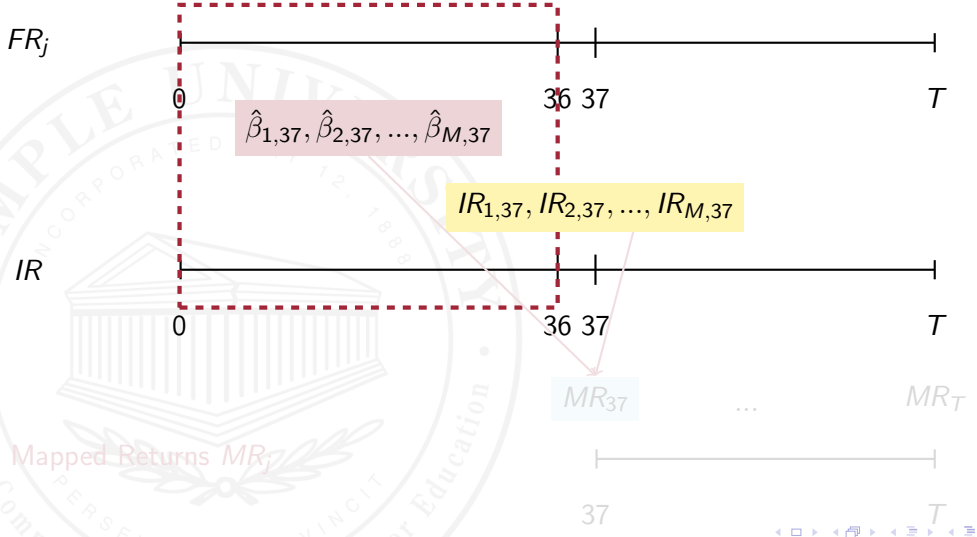

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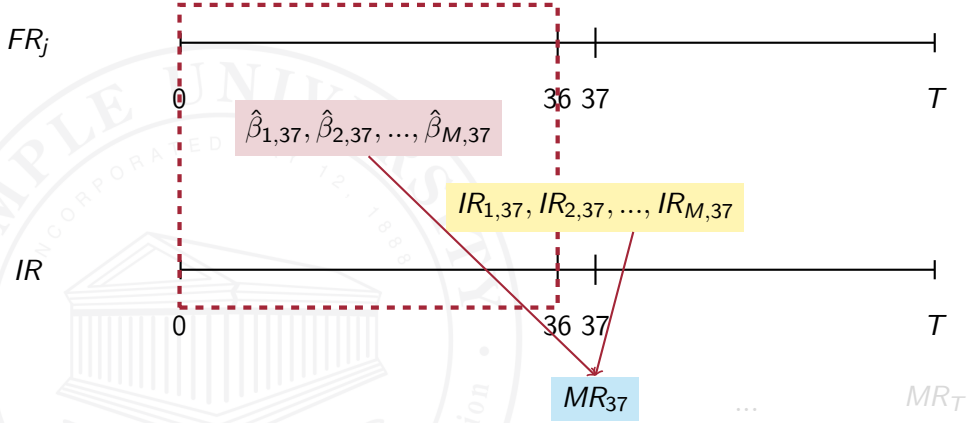
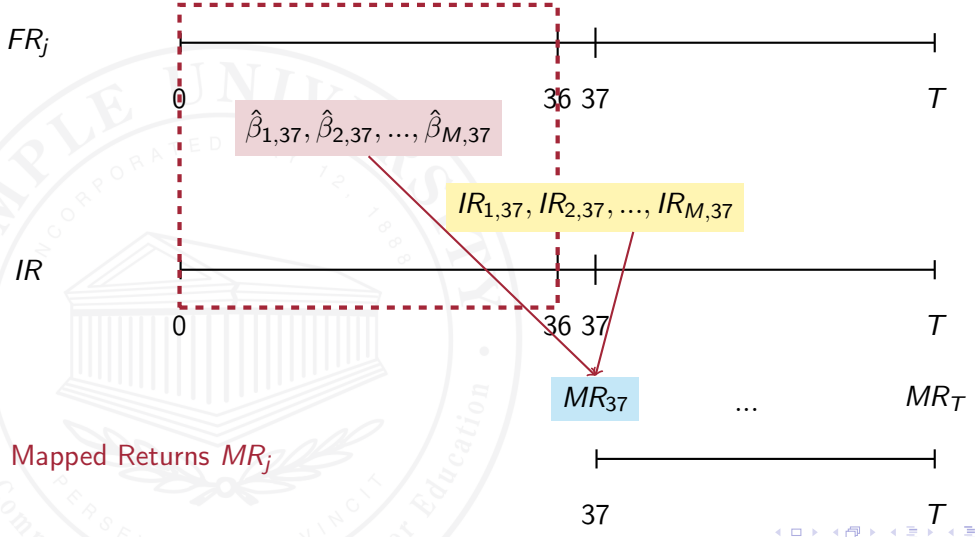
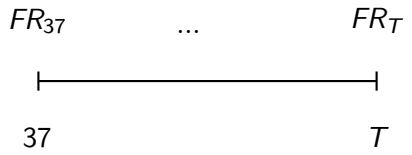


Rolling Window Period

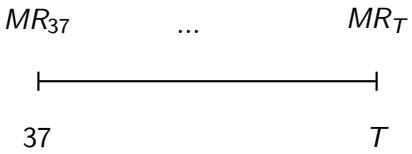


Basis Risk Measure I

Real Returns $\mathbf{FR}_j = (FR_{jt})_{t\in\{N+1,\dots,T\}}$



Mapped Returns $\mathbf{MR}_j = (MR_{jt})_{t\in\{N+1,\dots,T\}}$



- Basis risk for VA fund j (Ederington, 1979)

$$BR_j = \sqrt{1 - \mathrm{Corr}\left(\mathbf{FR}_j, \mathbf{MR}_j\right)^2}\,. \tag{3}$$

Hedging Instruments

- Optimal mapping strategies may vary across funds with different investment objectives
  - Lipper objective code (LOC): assigned based investment objectives
  - **Five instrument sets for each VA-underlying mutual fund j**
  - Constructed using 470 ETFs (62 LOC classes)

Set	$[k]$	Description	M
$I_j^{[1]}$	[1]	The representative ETF from fund j's LOC class.	1
$I_j^{[2]}$	[2]	All ETFs that have the same LOC as the mutual fund.	$[1, 56]$
$I_j^{[3]}$	[3]	All 62 representative ETFs.	62
$I_j^{[4]}$	[4]	All ETFs from fund j's LOC class, plus all representative ETFs.	$[62, 117]$
$I_j^{[5]}$	[5]	All 470 available ETFs.	470

  - Representative ETF: largest ratio of its average excess TNA compared to the average TNA for the LOC class, divided by the empirical standard deviation of the ETF's TNA

Basis Risk Measure II

- For fund mapping process of instrument set [2]-[5]: **LASSO regression**

$$\underset{(\beta_i)_{i\in I_j^{[k]}}}{\arg\min} \sum_{s=t-N}^{t-1} \left(FR_{js} - \sum_{i\in I_j^{[k]}} \beta_i IR_{is} \right)^2 + \lambda \sum_{i\in I_j^{[k]}} |\beta_i| \,. \tag{4}$$

- **The minimum level of basis risk for VA fund j across all instrument sets**

$$BR_j^* = \min_{k\in\{1,\ldots,5\}} BR_j^{[k]} \,. \tag{5}$$

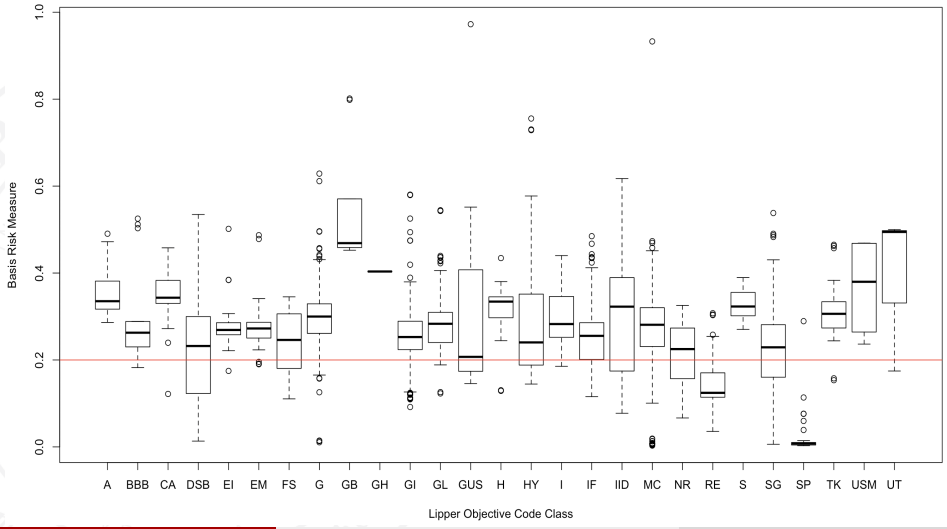
- Estimated share of the risk (i.e., standard deviation) of the underlying asset j that cannot be eliminated through hedging, even under the asset's most preferable instrument set

Data Selection

- Center for Research in Securities Prices (CRSP) Mutual Fund Database
- Sample Period: July 2009 - June 2019

Data Selection Steps	VA-underlying Mutual Funds	ETFs
Initial sample	5,673	2,772
Remove:		
Missing monthly returns data (7/2009 – 6/2019)	3,100	2,195
No representative ETF for its LOC class	451	N/A
Switched its LOC	232	106
One ETF as the proxy for risk-free asset	N/A	1
Final Sample	**1,890**	**470**

Result I - Basis Risk in Variable Annuities



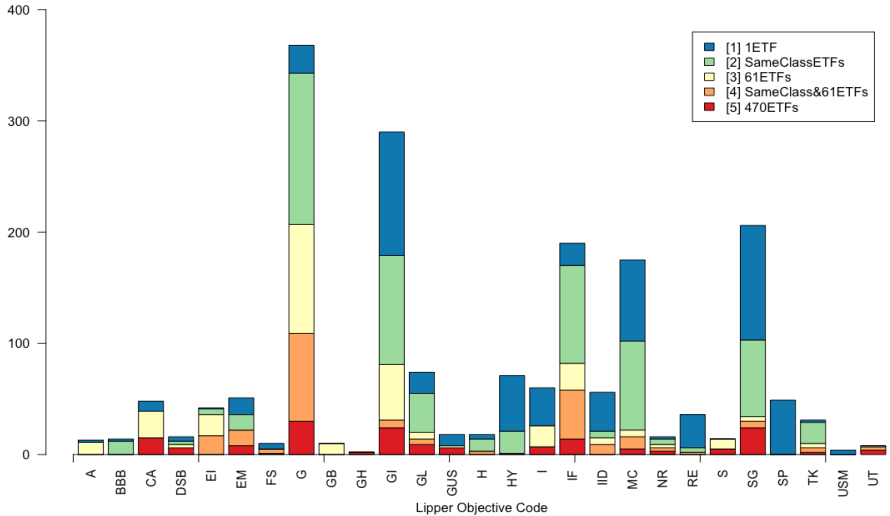
Result I - Basis Risk in Variable Annuities

	Sample Size	Mean	Std. Dev.	Min	Max	Weighted Avg.
(A) Total:						
	1,890	26.61%	10.49%	0.28%	97.25%	25.59%
(B) Grouped by Lipper Asset Code:						
Equity Funds	1,704	26.14%	9.79%	0.28%	93.28%	25.01%
Fixed Income Funds	186	30.96%	14.83%	7.74%	97.25%	30.30%
(C) Grouped by Index Fund Indicator:						
Pure Index Funds	183	9.60%	9.72%	0.28%	45.56%	5.27%
Other Index Funds	102	23.68%	10.78%	0.83%	53.46%	24.44%
Non-Index Funds	1,605	28.74%	8.57%	6.52%	97.25%	28.45%

Result I - Basis Risk in Variable Annuities

- (A): Basis risk is substantial and pervasive in U.S. VAs
  - On average a minimum level of 26.6% (weighted average: 25.6%)
  - Around 73.5% of VA funds have a basis risk between 20% and 40%
  - Around 7% of VA funds have a basis risk above 40%
- Figure: Basis risk present in nearly all LOC classes
  - 25 out of 27 LOC classes have a median basis risk above 20%
- (B): Fixed income funds face more basis risk than equity funds (31.0% vs. 26.1%)
- (C): Pure index funds contain substantially less basis risk: 9.6% on average
  - Actively managed VA funds are more difficult to track

Result II - Optimal Mapping Instrument Set



Result II - Optimal Mapping Instrument Set

$I^{[k]}$	Definition	Optimal	$\overline{BR}^{[k]}$	$\overline{BR}^{[k]}_{(w)}$
[1]	One representative ETF	32.22%	35.02%	33.49%
[2]	Same class ETFs	32.06%	30.63%	29.55%
[3]	All 62 representative ETFs	15.77%	31.86%	30.28%
[4]	Same class ETFs and 62 representative ETFs	11.16%	30.83%	29.73%
[5]	All 470 ETFs	8.78%	34.04%	33.33%
$\overline{BR}^{*}$, $\overline{BR}^{*}_{(w)}$			26.61%	25.59%

- A third of the VA funds are best mapped using instrument set [1]
- Instrument set [2] is preferable overall: 30.6%
- Instrument set [5] is optimal for only 8.8% of VA funds

Result III - Relating Basis Risk and Fund Characteristics

- Whether VA-underlying funds have distinct characteristics from other mutual funds?
  - All available mutual funds from June 2009 to July 2019
  - 1,890 VA funds and 8,027 non-VA funds
- How a fund's characteristics contribute to its estimated level of basis risk?

$$BR_j^* = \alpha_0 + \alpha_1 \mathrm{VA}_j + \alpha_2 \mathrm{FI}_j + \alpha_3 \mathrm{IX}_j + \alpha_4 \mathrm{Vol}_j \times \mathrm{FI}_j + \alpha_5 \mathrm{Vol}_j \times (1 - \mathrm{FI}_j) + \alpha_6 \mathrm{TNA}_j + e_j. \quad (6)$$

- BR_j^*: estimated basis risk level for mutual fund j
- VA_j: dummy variable that equals 1 if mutual fund j is a registered VA fund
- FI_j: dummy variable that equals 1 if mutual fund j is a fixed income fund
- IX_j: dummy variable that equals 1 if mutual fund j is a pure index fund
- Vol_j: standard deviation of mutual fund j's monthly returns over the sample period
- TNA_j: average month-end TNA (in \$ trillions) of mutual fund j

Result III - Relating Basis Risk and Fund Characteristics

Dependent variable	BR_j^*	Robust Standard Error
constant	0.3232***	(0.0064)
VA_j	-0.0094***	(0.0024)
FI_j	0.0755***	(0.0106)
IX_j	-0.1890***	(0.0051)
$Vol_j \times FI_j$	-7.3170***	(0.6066)
$Vol_j \times (1 - FI_j)$	-0.7206***	(0.1563)
TNA_j	-0.8141***	(0.2179)
N	9,913	
R^2	0.1755	

* p <0.05, ** p <0.01, *** p <0.001.

Result III - Relating Basis Risk and Fund Characteristics

- VA funds have a slightly lower basis risk than non-VA funds
- Pure index fund has a substantially less basis risk
- Fixed income funds contribute more to basis risk (7.5 percentage points)
  - However, guarantees written on fixed income funds are less valuable
- More volatile funds contain less basis risk (in both asset classes)

Robustness Checks

- Specification of fund returns
  - The use of *excess* returns produces a better fit: 26.6% vs. 28.0%
- Rolling window period
  - Try $N = 12, 24, 36, 48, 60$ months with a consistent testing period: July 2014 - June 2019
  - Basis risk tends to be lower for larger rolling window period
- Instrumenting with Futures
  - Include instrument set [6] with 71 futures contracts
  - ETFs are much better fund mapping instruments than futures contracts
- VA funds without representative ETF and VA funds with varying LOC
- Basis risk of market portfolio
  - Estimate basis risk for market portfolio (all 1,890 VA funds, weighted by TNA)
  - Minimum basis risk for market PF is lower than for average fund

Conclusion

- **Basis risk in VAs is substantial and pervasive**
  - More than a quarter of the risk cannot be eliminated through hedging
  - Pervasive across the funds' investment strategies and asset classes
- New insights of fund-mapping techniques through machine learning
  - Optimal instrument set combines data analytics and external information
  - Substantial improvement over ad-hoc fund mapping method
  - However: optimal mapping parameters vary greatly across funds
- How can VA providers reduce their exposure to basis risk?
  - Incentivize PHs to invest in index funds
  - Avoid funds with "exotic" investment strategies
  - Hedge with ETFs instead of futures contracts
  - Researchers: develop hedging strategies that explicitly account for basis risk

Future Work

- Potential interaction between basis risk and financial market scenarios
  - Basis risk can be higher during market downturns (Trottier et al., 2018)
  - When guarantees become most valuable (and insurers are most susceptible)
- ERM: how does basis risk impact life insurers' overall risk exposure?
- Device hedging strategies for VA guarantees that include basis risk
- Insurers usually bundle VA guarantees across many PHs
  - Guarantees vary by type, maturity, underlying asset, moneyness, etc.
  - How to minimize basis risk for VA *portfolios*?

Thank You!