Stable Value Fund Guarantees

An Asset Liability Risk Model

Work in Progress

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April 29, 2020

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How to lower the risk of a pension fund investment?

Lower Volatility $\Longleftrightarrow$ Lower Return

The main concern is not the daily volatility but

- business cycles and crisis.
- If buy and hold ✓, but what about emergency fund withdrawals?

Why not to synthesize an existing fund and make it less risky using structured finance?

Investment Stabilization Mechanism

Earning distribution schedule

Retain earnings in bull markets
Release past earnings in bear markets

(Miltersen and Persson, 2003) Optimal distribution schedule

Distribute Earnings $\iff$ Provision Earnings for Crisis

Asset	Liabilities
Initial Investment	Distributed Earnings
Market Returns	Non Distributed

(Kling et al., 2007) For fixed terms contracts.

Crediting Rate $\iff$ Reserves Surplus

Rational investor behaviour and disintermediation

(Wang et al., 2005) investor's Value-at-Risk-optimal policy. (Cheng et al., 2019) Optimal stopping time for rational investor (dynamic lapse model)

Cost of the guarantees

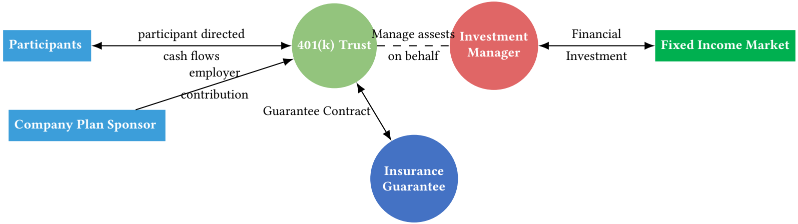
(Døskeland and Nordahl, 2008) Merton type Asset-Liability model

Stable Value Guarantees

Stable Value Fund Mechanism

Stable Value Fund Mechanism

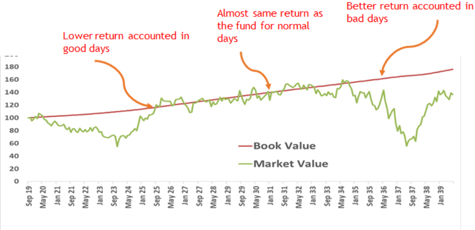
Stable value is a 401(k) retirement plan investment option regulated by US Employee Retirement Income Security Act (ERISA).



Stable value funds are synthesized version of a more volatile fund investment:

1. Return stabilization by Book Value accounting
2. The insurance company guarantees payments

Return Stabilization by Book Value Accounting



$$dB_t = B_t CR_t dt + \text{ Cash Flows}$$

$$CR_t = \left(\sqrt[1/\theta]{\frac{M_t}{B_t}} (1 + \text{yield}) - 1 \right)_+$$

M_t : fund's market value, B_t book value. Stable Value funds' duration θ are generally constant over time.

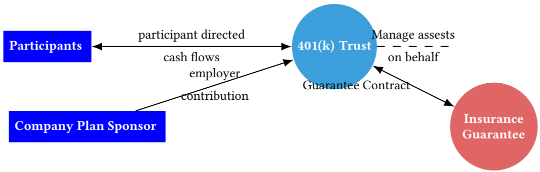
Book-to-Market convergence

If no cash flow activity $\rightarrow$ Book value converge to market value in θ years

$$E[B_{t+\theta}|\mathcal{F}_t] = E[M_{t+\theta}|\mathcal{F}_t]$$

Insurance Guarantees Book Value Payments

The insurance will make payment at the last resort*.



Last resort definition

The last participant requests its money back (M_t reaches 0)

$$\text{Insurance Payoff} = (B_\tau - M_\tau)_+$$

$$\tau = \min_t \{M_\tau = 0\}$$

Stable Value Guarantees

Asset Liability Model

Asset Liability Model

Assets	Liabilities
Fund's Asset (M_t)	Book Value (B_t)
Insurance guarantee	Sum of upcoming insurance premium

$$dB_t^p = dcf_t - B_t^p pdt + B_t^p CR_t dt$$
$$dM_t^p = dcf_t - B_t^p pdt + \dots$$

- cf is the sum of particpants' cash flows contribution/distributions
- p is the annualized premium for the insurance guarantee

$$dM_t^p = dcf_t - pB_t^p dt + M_t^p \,\text{drift}\, dt - M_t^p \theta dy_t$$

- Market Neutral: drift $= r_t + cs_t - \lambda_t$
- Historical World, if the yield does not have a term-structure: drift $= y_t - \lambda_t$

We define the yield y of the fund, as the average yield to worst of the fund components and the solution to: $\left.\frac{\partial M_t^p(r)}{\partial r}\right|_y = M_t^p(r) \times \theta$

λ_t is the default/credit quality migration rate of the fund's portfolio.

Market-to-Book value ratio: $MBV_t = \frac{M_t}{B_t}$

$$\frac{dMBV_t}{MBV_t} = (y_t - \lambda_t - CR_t)\, dt - \theta dy_t - \frac{1}{MBV_t} \times p \times dt - (1 - \frac{1}{MBV_t}) \times d(\zeta_t - [\zeta]_t)$$

Stable Value Guarantees

Optimal Stabilization Mechanism

Optimal Stabilization Mechanism

- How the Book Value B_t should be designed, optimally?
- Is the crediting rate formula good enough

$$CR_t = \max\left(floor, \left(y_t + spread - \frac{1}{DAF \times \theta} \ln\left(\frac{M_t}{B_t}\right)\right)\right)$$

Better crediting rate formula $\Longleftrightarrow$ Higher insurance premium

$$p = \gamma \times \frac{E\left[(B_\tau - M_\tau)_+\right]}{E\left[\int_0^\tau B_t dt\right]}$$

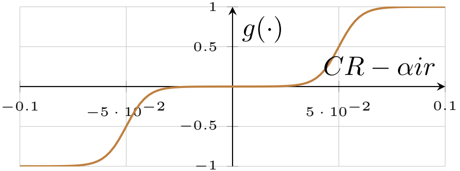
γ: risk aversion factor.

Crediting rate formula	Theoretical annual premium
Standard formula $+0.25\%$	0.28%
Standard formula	0.20%
Standard formula -0.5%	0.12%
Standard formula -1%	0.10%

Lower crediting rate formula $\iff$ Less contributions/ higher withdrawals

$$dcf_t = B_t \times d\zeta_t$$
$$d\zeta_t = g\left(CR_t - \alpha ir\left(\frac{M_t}{B_t}\right)\right)dt + h\left(\frac{M_t}{B_t}\right)dW_t$$

- ζ: the cash flow rate
- $g(\cdot)$: Dynamic cash flows in a *double-S-shape* form (Eling and Kochanski, 2013)



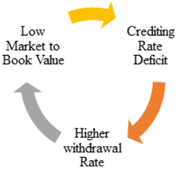
- $\alpha ir(\frac{M_t}{B_t})$: alternative investment rate.

Disintermediation risk

For simplicity, we call $B_t = B_t^0, M_t = M_t^0$.

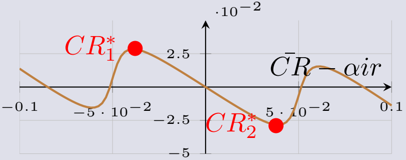
$$U(t, MBV_t, y_t) = \sup_{CR} E\left[\frac{1}{M_T}\left(B_T - \gamma\left(B_T - M_T\right)_+\right)|\mathcal{F}_t\right]$$

$$\max_{CR}\{\partial_2 U \times (\ -CR + y + \lambda + \left(\frac{B_t}{M_t} - 1\right)g(CR - \alpha ir(MBV_t))$$



Case Study: $MBV \ll 1$

$\partial_2 U > 0$	CR_1^*
$\partial_2 U < 0$	Few scenarios



To mitigate any disintermediation risk, the optimal crediting rate (CR_1^*) would be just slightly lower than the alternative investment rate.

Conclusion:

- We developed an asset-liability model, tailor-made to stable value funds.
- We observed how the choice of crediting rate formula can mathematically and financially mitigate/increase the disintermediation risk.
- We observed the relationship between crediting rate formula and the insurance cost.

Prospects:

- Study further the relationship between crediting rate policy and insurance premium.
- Study further the optimality of the market standard formula.

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