Holistic Aggregation and Allocation Principle

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Motivation

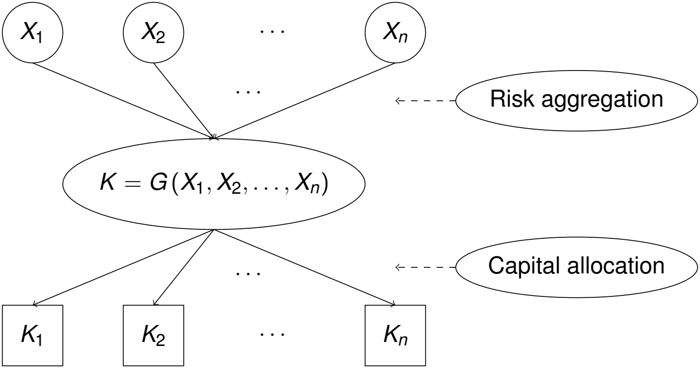

Figure: Two-step procedure – risk aggregation and capital allocation

Motivation

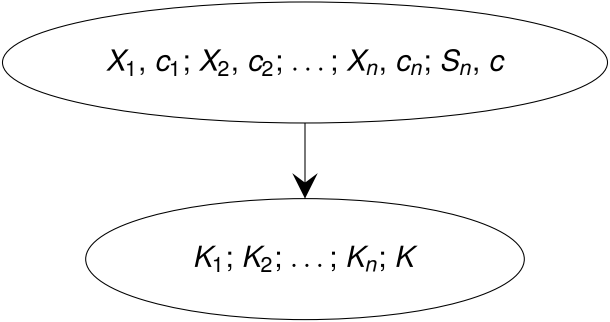

Figure: One-step procedure - holistic allocation

Motivation

Source: What is a balanced flue?

Figure: Balancing game between capital management dept. and risk management dept.

Motivation

	Capital management	Risk management
Objective function	C(K)	R(S; K)
Goal	Minimize C(K)	Minimize R(S; K)
Minimizer	K^C	K^R
Example	Cost of capital rK	Shortfall $(S-K)_+$
	Targeted capital $(K-c)^2$	Weighted deviation $(S-K)^2h(S)$

Table: Different objective in capital/risk management dept.

Motivation

- Pareto optimality
  Suppose there are n lines of business in a corporate

	Capital management	Risk management	
1-st LOB/Risk	$\inf C_1(K_1)$	$\inf R_1(K_1; X_1)$	X_1
2-nd LOB/Risk	$\inf C_2(K_2)$	$\inf R_2(K_2; X_2)$	X_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n-th LOB/Risk	$\inf C_n(K_n)$	$\inf R_n(K_n; X_n)$	X_n
Corporate	$\inf C(K)$	$\inf R(K; S)$	S

Figure: Multiobjective capital allocation – competing interests and conflicting priorities

1. Overview
   - Motivation
   - Mathematical Formulation
   - Holistic Principle: Unconstrained Case

Mathematical Formulation

Holistic Principle

The holistic capital allocation is the set of real numbers $(K_1^H, \ldots, K_n^H)$ given by the unique minimizer of the following multi-objective optimization problem:

$$\inf_{(K_1,K_2,\ldots,K_n)\in\mathcal{K}} \sum_{i=1}^{n} \nu_i C_i\left(K_i\right) + \sum_{i=1}^{n} \omega_i R_i\left(K_i; X_i\right) + \nu C\left(K\right) + \omega R\left(K; S\right),$$

where $K = \sum_{i=1}^{n} K_i$ and $\mathcal{K} \subseteq \mathbb{R}^n$ is a non-empty set of admissible set; ν_i, ω_i, ν, and ω are non-negative weights, while at least one of them is positive; C, R, C_i, R_i are real-valued functions.

1 Overview

- Motivation
- Mathematical Formulation
- Holistic Principle: Unconstrained Case

Holistic Principle: Unconstrained Case

Target Aiming and Weighted Deviation Case

Consider the holistic capital allocation $\left(K_1^H, K_2^H, \ldots, K_n^H\right)$ such that:

$$\inf_{(K_1,K_2,\ldots,K_n)\in\mathbb{R}^n} \sum_{i=1}^{n} \nu_i \left(c_i - K_i\right)^2 + \sum_{i=1}^{n} \omega_i \mathbb{E}\left[\left(X_i - K_i\right)^2 h_i\left(X_i\right)\right]$$
$$+ \nu\left(c - K\right)^2 + \omega\mathbb{E}\left[\left(S - K\right)^2 h\left(S\right)\right].$$

Holistic Principle: Unconstrained Case

Optimal Solution

$$
\begin{aligned}
K^H &= \overline{K} + A\left(\sum_{r=1}^{n} \overline{K}_r - \overline{K}\right), \\
K_i^H &= \overline{K}_i - B_i\left(\sum_{r=1}^{n} \overline{K}_r - \overline{K}\right),
\end{aligned}
$$

Holistic Principle: Unconstrained Case

Optimal Solution

for any $i = 1, 2, \ldots, n$, where

$$\overline{K}_i = \alpha_i K_i^C + \beta_i K_i^R, \quad \overline{K} = \alpha K^C + \beta K^R,$$

$$K_i^C = c_i, \quad K_i^R = \rho_i(X_i) = \mathbb{E}\left[X_i h_i(X_i)\right],$$
$$K^C = c, \quad K^R = \rho(S) = \mathbb{E}\left[S h(S)\right],$$

$$A = \frac{\frac{1}{\nu+\omega}}{\frac{1}{\nu+\omega} + \sum_{r=1}^{n} \frac{1}{\nu_r+\omega_r}}, \quad B_i = \frac{\frac{1}{\nu_i+\omega_i}}{\frac{1}{\nu+\omega} + \sum_{r=1}^{n} \frac{1}{\nu_r+\omega_r}},$$

$$\alpha_i = \frac{\nu_i}{\nu_i + \omega_i}, \quad \beta_i = \frac{\omega_i}{\nu_i + \omega_i}, \quad \alpha = \frac{\nu}{\nu + \omega}, \quad \beta = \frac{\omega}{\nu + \omega}.$$

Holistic Principle: Unconstrained Case

Optimal Aggregate Capital

$$\overline{K} = \arg\min \left\{ \nu C(K) + \omega R(K; S) \right\} = \alpha K^C + \beta K^R$$

Optimal Standalone Capital

$$\overline{K}_i = \arg\min \left\{ \nu_i C_i(K_i) + \omega_i R_i(K_i; X_i) \right\} = \alpha_i K_i^C + \beta_i K_i^R$$

Diversification Benefit

$$\sum_{r=1}^{n} \overline{K}_r - \overline{K}$$

Holistic Principle: Unconstrained Case

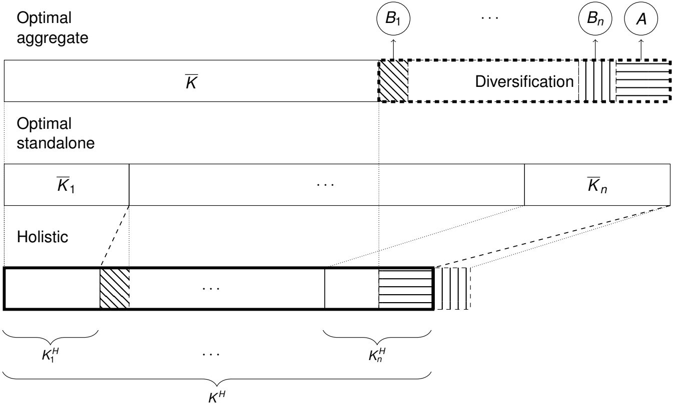

Figure: Holistic aggregate and allocated capitals – relationships with optimal aggregate and standalone capitals

Holistic Principle: Unconstrained Case

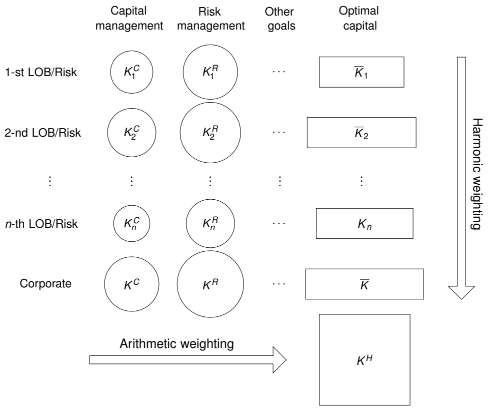

Figure: Holistic aggregate and allocated capitals – balancing of optimal capitals

Holistic Principle: Unconstrained Case

Properties of Optimal Solution

- Monotonic property I
- Monotonic property II
- Convergence property

Reference

- Chong, W. F., Feng, R., & Jin, L. (2020). Holistic Principle for Risk Aggregation and Capital Allocation. Available at SSRN 3544525.

Thanks for your listening!